Benchmarking quantum error-correcting codes using Monte Carlo sampling

Michael Vasmer, TQCI seminar, 24/06/25

Motivation

- Quantum error correction (QEC) will be crucial for building large-scale and reliable quantum computers
- Impressive recent progress in QEC experiments [RBL+21, AAA+24, BEG+24, …] **but** these experiments are difficult, time-consuming, and expensive
- Benchmarking QEC codes *in silico* allows us to evaluate code performance quickly and guide the next generation of experiments

Recap: stabilizer code decoding

- QEC code defined by commuting Pauli operators (stabilizers)
- We measure the stabilizers to detect errors
- **Error syndrome**: list of stabilizer measurement outcomes
- Classical algorithm (**decoder**) decides on a recovery operator given a syndrome
- Decoding is successful if the recovery is equivalent to the error (up to a stabilizer)

Two figures of merit

- **Error threshold**: maximum error rate allowed
- **"Teraquop" footprint**: amount of resources needed to do something useful

Error threshold

- Specified for
  - Family of codes of growing size, parameterised by D
  - Noise model parameterised by p
  - Decoder
- For error rates $p < p^*$ the logical error rate $p_{\text{fail}} \sim p^D$
- **Exponential suppression** of errors below the threshold p^*

Estimating the error threshold

- Pick some codes from your family
    - Surface codes of size $D \in \{7, 11, 15, 21, 29\}$
- For each code estimate the logical error rate for a variety of noise parameters p
    - Sample an error from the noise model
    - Compute syndrome and feed to decoder
    - Check whether recovery operator is equivalent to the error
    - Repeat many times ($\sim$ 10,000) and count the number of failures

Estimating the error threshold

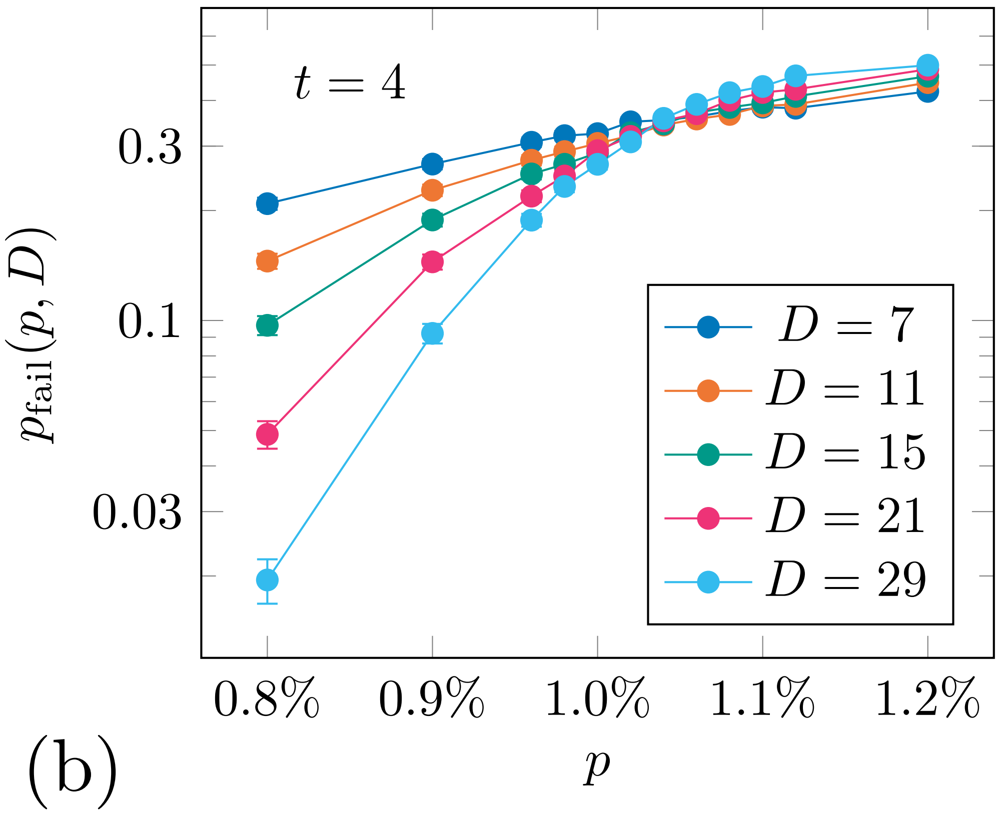

Figure from [KV22]

What noise models can we sample from efficiently?

- Pauli noise (Gottesman-Knill theorem) [Got98, AG04, Gid21]
- Erasure and leakage (depends on the model) [BNB19, SJC+23, GYA+24]
- Some noise models with Clifford errors [Bom18, BKS21, SVB22]
- Coherent errors (2D surface codes) [BEK+18]
- Local noise (2D surface code) [DP17]

Teraquop footprint

- Fix the noise parameter p
- How many physical qubits are required to reach a logical error rate of 10^{-12} ?
  - Means we can implement $\sim 10^{12}$ gates
  - Number of gates necessary for utility-scale (quantum chemistry, factoring, ...)

Estimating the teraquop footprint

- Below threshold we expect $p_{\text{fail}} = a(p/p^*)^{bD^c}$
- Estimate logical error rates below threshold and fit the parameters a, b, and c
- E.g. for surface code $a \approx 0.03, b \approx 0.5, c \approx 1, p^* \approx 1\%$ [FMM+12]
  - For $p = 10^{-3}$ we need $D = 21$ to reach $p_{\text{fail}} = 10^{-12}$
  - This corresponds to $2D^2 = 882$ physical qubits

Which figure of merit is most important?

- **Threshold**
  - If you are above threshold then your quantum computer won't work at all
  - Only one experiment with convincing sub-threshold behaviour [AAA+24]
- **Footprint**
  - If your footprint is too large then your quantum computer will be too large and expensive to build

Limits of Monte Carlo sampling?

- Threshold plots $\neq$ proof of exponential error suppression
- Most QEC simulations consider relatively high logical error rates (10^{-1} to 10^{-6})
- How can we be sure of reaching logical error rates of 10^{-12} ?
  - Naïve Monte Carlo would need $\gg 10^{12}$ trials!
  - Proofs possible in special cases
  - More sophisticated Monte Carlo [BV13], only for surface code so far

Food for thought: error floors

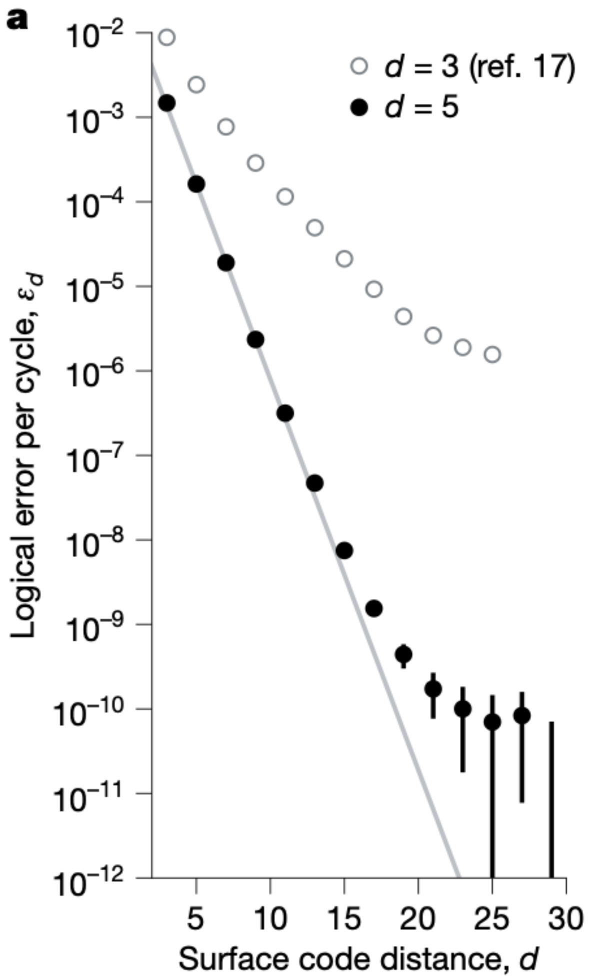

Figure from [AAA+24]

Conclusion

- Monte Carlo sampling is widely used to benchmark QEC codes, for a variety of noise models and decoders
- Two key figures of merit: error threshold and (teraquop) footprint
- It is difficult to simulate device noise exactly and to probe the low logical error rate regime

References

- [RBL+21] C. Ryan-Anderson, J. G. Bohnet, K. Lee et al., "Realization of Real-Time Fault-Tolerant Quantum Error Correction", *Physical Review X* (2021)
- [AAA+24] R. Acharya, D. A. Abanin, L. Aghababaie-Beni et al., "Quantum error correction below the surface code threshold", *Nature* (2024)
- [BEG+24] D. Bluvstein, S. J. Evered, A. A. Geim et al., "Logical quantum processor based on reconfigurable atom arrays", *Nature (2024)*
- [KV22] A. Kubica, M. Vasmer, "Single-shot quantum error correction with the three-dimensional subsystem toric code", *Nature Communications* (2022)

References

- [Got98] D. Gottesman, "The Heisenberg Representation of Quantum Computers", *Group22: Proceedings of the XXII International Colloquium on Group Theoretical Methods in Physics* (1998)
- [AG04] S. Aaronson, D. Gottesman, "Improved simulation of stabilizer circuits", *Physical Review A* (2004)
- [Gid21] C. Gidney, "Stim: a fast stabilizer circuit simulator", *Quantum* (2021)
- [Bom18] H. Bombín, "Transversal gates and error propagation in 3D topological codes", *arXiv preprint* (2018)

References

- [BKS21] M. E. Beverland, A. Kubica, K. M. Svore, "Cost of Universality: A Comparative Study of the Overhead of State Distillation and Code Switching with Color Codes", *PRX Quantum* (2021)
- [SVB22] T. R. Scruby, M. Vasmer, D. E. Browne, "Non-Pauli errors in the three-dimensional surface code", *Physical Review Research* (2022)
- [BEK+18] S. Bravyi, M. Englbrecht, R. König, N. Peard, "Correcting coherent errors with surface codes", *npj Quantum Information* (2018)
- [BNB19] N. C. Brown, M. Newman and K. R. Brown, "Handling leakage with subsystem codes", *New Journal of Physics* (2019)

References

- [SJC+23] K. Sahay, J. Jin, J. Claes et al., "High-threshold codes for neutral-atom qubits with biased erasure errors", *Physical Review X* (2023)
- [GYA+24] S. Gu, Y. Vaknin, A. Retzker, A. Kubica, "Optimizing quantum error correction protocols with erasure qubits", *arXiv preprint* (2024)
- [DP17] A. Darmawan, D. Poulin, "Tensor-Network Simulations of the Surface Code under Realistic Noise", *Physical Review Letters* (2017)
- [FMM+12] A. G. Fowler, M. Mariantoni, J. M. Martinis, A. N. Cleland. "Surface codes: Tow Ards practical large-scale quantum computation", *Physical Review A* (2012)
- [BV13] S. Bravyi, A. Vargo, "Simulation of rare events in quantum error correction", *Physical Review A* (2013)